Journal of Engineering Mathematics **49:** 133–147, 2004.

Interaction of free-surface waves with floating flexible strips

A.J. HERMANS
TU Delft, Faculty of Electrical Engineering, Mathematics and Computer Science, Department of Applied Mathematical Analysis, Mekelweg 4, 2628 CD Delft, The Netherlands (e-mail: a.j.hermans@its.tudelft.nl)

Received 25 May 2003; accepted in revised form 7 October 2003

Abstract. The method developed by the author to derive a set of algebaic equations to solve the interaction of free-surface waves with a single floating rigid or flexible two-dimensional platform with small draft is extended to the case that the platform consists of strips with different constant flexural rigidity and mass. The method is based on the application of Green's theorem, with a specific choice of the Green function to arrive at a differential-integral equation along the platform. This equation can be solved exactly by means of superposition of exponential functions, a standard method to solve a set of linear differential equations. After integration with respect to the space coordinate the residue theorem leads to both the dispersion relation along each individual strip and an algebraic equation for the coefficients. Due to very fast convergence with respect to the number of coefficients taken into account the series are truncated. Depending on the water-depth, in each series three to ten terms are taken into account. Results are shown for a structure consisting of several strips that are tightly connected and for disjoint strips. In the latter case the computation of the water level between the strips is also computed. The water level and the reflection and transmission coefficients are not unknowns in the algebraic equation, but are computed afterwards by means of Green's theorem.

Key words: boundary-integral equations, flexible platform, free-surface flows, hydroelasticity, wave diffraction

1. Introduction

One of the concepts for future floating airports is the construction of an artificial floating island consisting of a flexible mat-like structure. One wants to make it as light as possible, hence very flexible. The deflection of the platform due to incident waves increases if the value of the flexural rigidity decreases. It is of interest to study the effect of a platform consisting of a strip of high rigidity attached to an area of low rigidity. We present a method that makes it possible, in a rather simple way, to study this effect. With this approach it is also possible to investigate the effect of disjoint strips.

We consider the two-dimensional interaction of a plane wave with a floating flexible platform. The platform consist of several strips and its draft is assumed small. The water depth is finite. The problem of wave interaction with a single strip has been studied by several authors. Most approaches are based on the classical method of expanding the velocity potential by means of eigenfunctions in two separate regions. For the fixed rigid platform with finite draft Mei and Black [1] published this method in 1969. They solved the remaining equations by means of a variational approach. For the small-draft case it is shown by Linton [2] that the algebraic system obtained by this approach can be solved analytically. It is shown by Linton and Chung [3] that this approach is applicable to solve the semi-infinite flexible plate case, with waves entering either from the water region or the platform region. They obtain perfect agreement with results obtained originally, by means of the Wiener-Hopf technique, by Evans and Davies [4]. Chung and Linton [5] apply their residue technique to the case of a finite gap as well. The approach of Linton becomes more complicated than the one presented here, if it is applied to the case of multiple strips with different elastic coefficients.

Recently Evans and Porter [6] studied the case of a narrow straight-line crack separating two semi-infinite thin elastic plates floating on water of finite depth. Their formulation is based on an expansion in non-orthogonal eigenfunctions. The method is very efficient; however, it is restricted to the case of plane waves incident at the crack. In our paper we also take into account strips of limited size with different flexural rigidity and we allow the complete field, incident travelling and incident evanescent modes, to be present near the connection or crack. For an overview of recent related work on sea-ice interaction we refer to Evans and Porter [6] and Squire *et al.* [7].

In an earlier paper Hermans [8] derived a formulation based on Green's theorem. In that paper the solution is obtained by means of two numerical approaches based on a panel method and the expansion in dry eigenmodes. Meanwhile this method has been applied to a three-dimensional platform by Guéret [9]. For the deep-water case Hermans [10] has shown that an approximate solution can be obtained in the two-dimensional case by means of a superposition of exponential functions. The effect of the continuous spectrum is taken into account iteratively. Recently Hermans [11] applied the same approach to the single-platform case. In principle this approach takes care of all the wave modes; however, due to fast convergence the number of modes taken into account may be truncated after a few terms. This problem can also be solved by Linton's analytical approach.

Here we extend our approach to the multi-strip problem. It then becomes clear that our method is very efficient for solving this problem. At each strip the deflection is written as a series of exponential functions. Because the physical parameters at each strips are assumed to be constants, the integration in the differential-integral equations with respect to the *x*-coordinates can be carried out at each individual strip. Application of Cauchy's residue theorem leads to the dispersion relations at the strips and to a set of algebraic equations for the constant amplitudes. The structure of the algebraic equation is suitable for solving it by means of a method similar to the one used by Linton; however, we solve the infinite system of algebraic equations by a truncation method.

We show results for connected and disconnected multiple strips. An advantage of this approach is that the matrix equation only contains the unknown coefficients at the strips. The free-surface water-wave heights are computed afterwards by means of Green's theorem. The formulation presented is for the case that the incident waves are travelling perpendicular to the strips. In the appendix of [11] the equations for a strip with waves at oblique angles are given.

2. Mathematical formulation for the zero-draft platform

In this section we derive the general formulation for the diffraction of waves by a flexible platform of general geometric shape. The fluid is inviscid and incompressible, so we introduce the velocity potential $\mathbf{V}(\mathbf{x}, t) = \nabla\Phi(x, t)$, where $\mathbf{V}(\mathbf{x}, t)$ is the fluid velocity vector. Hence $\Phi(\mathbf{x}, t)$ is a solution of the Laplace equation

$$\Delta\Phi = 0 \quad \text{in the fluid,} \tag{1}$$

together with the linearized kinematic condition, $\Phi_z = \tilde{w}_t$, and dynamic condition, $p/\rho = -\Phi_t - g\tilde{w}$, at the mean water surface $z = 0$, where $\tilde{w}(x, y, t)$ denotes the free-surface elevation, and ρ is the density of the water. The linearized free-surface condition outside the platform, $z = 0$ and $(x, y) \in \mathcal{F}$, becomes:

$$\frac{\partial^2 \Phi}{\partial t^2} + g\frac{\partial \Phi}{\partial z} = 0. \tag{2}$$

The platform is assumed to be a thin layer at the mean free-surface $z = 0$, which seems to be a good model for a shallow-draft platform. The platform is modelled as an elastic plate with zero thickness. To describe the deflection $\tilde{w}(y, y, t)$, we apply the isotropic thin-plate theory, which leads to an equation for $\tilde{w}$ of the form

$$m(x, y)\frac{\partial^2 \tilde{w}}{\partial t^2} = -\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\left(D(x, y)\left(\frac{\partial^2 \tilde{w}}{\partial x^2} + \frac{\partial^2 \tilde{w}}{\partial y^2}\right)\right) + p|_{z=0}, \tag{3}$$

where $m(x, y)$ is the mass of unit area of the platform while $D(x, y)$ is its equivalent flexural rigidity. We differentiate (3) with respect to t and use the kinematic and dynamic condition to arrive at the following equation for Φ at $z = 0$ in the platform area $(x, y) \in \mathcal{P}$:

$$\left\{\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\left(\frac{D(x, y)}{\rho x^2}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\right) + \frac{m(x, y)}{\rho g}\frac{\partial^2}{\partial t^2} + 1\right\}\frac{\partial \Phi}{\partial z} + \frac{1}{g}\frac{\partial^2 \Phi}{\partial t^2} = 0. \tag{4}$$

The edges of the platform are free of shear forces and moment. We assume that the flexural rigidity is constant along the edge and its derivative normal to the edge equals zero. Also, we assume that the radius of curvature, in the horizontal plane, of the edge is large. Hence, the edge may be considered to be straight locally. We then have the following boundary conditions at the edge:

$$\frac{\partial^2 \tilde{w}}{\partial n^2} + \nu\frac{\partial^2 \tilde{w}}{\partial s^2} = 0 \text{ and } \frac{\partial^3 \tilde{w}}{\partial n^3} + (2 - \nu)\frac{\partial^3 \tilde{w}}{\partial n \partial s^2} = 0, \tag{5}$$

where ν is Poisson's ratio, n is in the normal direction, in the horizontal plane, along the edge and s denotes the arc-length along the edge. At the bottom of the fluid region $z = -h$ we have:

$$\frac{\partial \Phi}{\partial z} = 0. \tag{6}$$

We assume that the velocity potential is a time-harmonic wave function, $\Phi(\mathbf{x}, t) = \phi(\mathbf{x})e^{i\omega t}$. We introduce dimensionless coordinates and parameters in the following way:

$$\mathbf{x}' = \frac{\mathbf{x}}{L},\ h' = \frac{h}{L},\ K = \frac{\omega^2 L}{g}, \mu = \frac{m\omega^2}{\rho g},\ \mathcal{D} = \frac{D}{L^4 \rho g}.$$

In a practical situation the total lengh L of the platform is a few thousand meters. After dropping the primes we obtain at the free surface

$$\frac{\partial \phi}{\partial z} - K\phi = 0 \tag{7}$$

and at the plate

$$\left\{\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\left(D(x, y)\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\right) - \mu(x, y) + 1\right\}\frac{\partial \phi}{\partial z} - K\phi = 0. \tag{8}$$

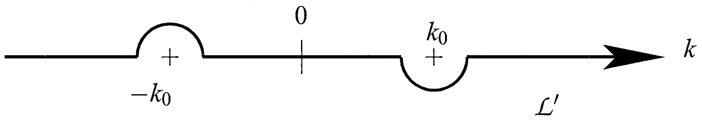


Figure 1. Contour of integration.

The potential of the undisturbed incident wave is given by:

$$\phi^{\mathrm{inc}}(\mathbf{x}) = \frac{g\zeta_\infty}{\mathrm{i}\omega}\frac{\cosh(k_0(z+h))}{\cosh(k_0 h)}\exp\{\mathrm{i}k_0(x\cos\beta + y\sin\beta)\}, \tag{9}$$

where ζ_∞ is the wave height in the original coordinate system, ω the frequency, while the wave number k_0 is the positive real solution of the dispersion relation,

$$k_0\tanh(k_0 h) = K, \tag{10}$$

for finite water depth. We restrict ourselves to the case of normal incidence, $\beta = 0$. In [11] it is shown that the extension to oblique waves can be done easily.

To obtain an integral equation for the deflection $\tilde{w}(x, y, t) = \mathfrak{Re}\left[\zeta_\infty w(x, y)\mathrm{e}^{-\mathrm{i}\omega t}\right]$ of the platform ([8] and [11]) it is very convenient to apply Green's theorem, making use of the Green's function, $\mathcal{G}(\mathbf{x};\xi)$, that fulfills the free-surface boundary condition (7). Application of Green's theorem in the fluid domain leads to the following expression for the potential function,

$$4\pi\phi(\mathbf{x}) = 4\pi\phi^{\mathrm{inc}}(\mathbf{x}) + \int_{\mathcal{P}}\left(K\phi(\xi) - \frac{\partial\phi(\xi)}{\partial z}\right)\mathcal{G}(\mathbf{x};\xi)\mathrm{d}S. \tag{11}$$

In the two-dimensional case, (x, z)-plane, the expression for the total potential becomes:

$$2\pi\phi(x,z) = 2\pi\phi^{\mathrm{inc}}(x,z) + \int_{\mathcal{P}}\left(K\phi(\xi,0) - \frac{\partial\phi(\xi,0)}{\partial z}\right)\mathcal{G}(x,z;\xi,0)\mathrm{d}\xi. \tag{12}$$

At $z = \zeta = 0$ the two-dimensional Green's function for finite water depth, obeying the radiation condition, has the form:

$$\mathcal{G}(x,0;\xi,0) = -\int_{\mathcal{L}'}\frac{\cosh kh}{k\sinh kh - K\cosh kh}\mathrm{e}^{\mathrm{i}k(x-\xi)}\mathrm{d}k \tag{13}$$

and the three-dimensional version has the form:

$$\mathcal{G}(x,y,0;\xi,\eta,0) = -2\int_0^\infty \frac{k\cosh kh}{k\sinh kh - K\cosh kh}J_0(kR)\mathrm{d}k \tag{14}$$

The contour of integration $\mathcal{L}'$ in (13) is given in Figure 1, where k_0 is defined in (10). The contour of integration in (14) is the right-hand part of $\mathcal{L}'$. It is chosen such that the radiation condition is fulfilled. R is the horizontal distance, so $R^2 = (x-\xi)^2 + (y-\eta)^2$.

Application of condition (8) for the potential function at the platform leads, in the two-dimensional case, to the following equation for the defection $w(x)$,

$$\begin{aligned} &2\pi\left\{1 - \mu(x) + \frac{\mathrm{d}^2}{\mathrm{d}x^2}\mathcal{D}(x)\frac{\mathrm{d}^2}{\mathrm{d}x^2}\right\}w(x) \\ &\quad + K\int_{\mathcal{P}}\mathcal{G}(x,0;\xi,0)\left\{\mu(\xi) - \frac{\mathrm{d}^2}{\mathrm{d}\xi^2}\mathcal{D}(\xi)\frac{\mathrm{d}^2}{\mathrm{d}\xi^2}\right\}w(\xi)\mathrm{d}\xi = 2\pi\mathrm{e}^{\mathrm{i}k_0 x}. \end{aligned} \tag{15}$$

This equation was derived in [8].

3. Distribution of piecewise constant parameters

We consider a two-dimensional platform consisting of I connected strips. The flexural rigidity and mass are piece-wise constant functions. The coefficients can be written as

$$\mathcal{D}(x) = \mathcal{D}_i, \mu(x) = \mu_i, \text{ where } l_{i-1} < x < l_i, \text{ for } i = 0, \dots I. \tag{16}$$

l_{i-1} is the coordinate of the left edge of the *i*-th elastic strip and l_i is the coordinate of its right edge. In normalized coordinates $l_I = 1$. In this case the differential-integral equation (15) can be reduced to a set of algebraic equations for the coefficients of the following expansions.

In each interval we assume that the solution can be expressed as the sum of exponential functions truncated at $N + 2$ terms

$$w(x) = \sum_{n=0}^{N+1} \left(a_{i,n} e^{i\kappa_{i,n}(x-l_{i-1})} + b_{i,n} e^{i\kappa_{i,n}(x-l_i)} \right) \text{ for } l_{i-1} < x < l_i. \tag{17}$$

If we consider κ's with either real positive values or, if they are complex, with positive imaginary part, then the first part of expression (17) expresses modes travelling and evanescent to the right. The second part then describes modes travelling and evanescent to the left. We have introduced $2I(N + 2)$ unknown coefficients $a_{i,n}, b_{i,n}$, but also $I(N + 2)$ unknown κ's. It will be shown that $\kappa_{i,n}$ obeys the dispersion relation of the *i*'s strip:

$$(\mathcal{D}_i \kappa_{i,n}^4 - \mu_i + 1)\kappa_{i,n} \tanh(\kappa_{i,n} h) = K. \tag{18}$$

At the edges $x = 0$ and $x = 1$ of the platform we have zero bending moment and force, while at the connection points $x = l_i$ we have continuity of the deflection and of its first derivative while the bending moment and force are continuous as well. The last two conditions result in continuity of $\mathcal{D}w''$ and $\mathcal{D}w'''$. One may also choose other constructions for the connection of the strips. In the case that the strips are free to move with respect to each other we have zero shear force and bending moment at both edges. It is also possible to leave a gap between the strips. All these problems lead to $4I$ relations between the coeficients of the expansions. In the next section we will derive the dispersion relation to determine the κ's and $2IN$ linear equations for the expansion coefficients. The integral in the expression for the deflection (15) consists of I integrals, so it is convenient to study one of them first. We consider the integral over the *i*th interval

$$I_i = K \int_{l_i-1}^{l_i} \mathcal{G}(x, 0; \xi, 0) \left\{ \mu(\xi) - \frac{d^2}{d\xi^2} \mathcal{D}(\xi) \frac{d^2}{d\xi^2} \right\} w(\xi) d\xi, \tag{19}$$

for $0 < x < L$. This means that we study one strip first and its infuence on the other strips. We distinguish three cases, $x < l_{i-1}, l_{i-1} < x < l_i$ and $x > l_i$. The second case gives, besides a contribution to the matrix equation, the dispersion relation. The two other cases lead to contributions to the matrix equation only.

3.1. Dispersion relations and matrix equation

We insert the expression for the Green's function (13) in (19) and change the order of integration to obtain the following expression

$$I_i = K \int_{\mathcal{L}'} \frac{\cosh kh e^{ikx}}{k \sinh kh - K \cosh kh} \left\{ \int_{l_{i-1}}^{l_i} \left\{ \mu_i - \mathcal{D} \frac{d^4}{d\xi^4} \right\} w(\xi) e^{-ik\xi} d\xi \right\} dk, \tag{20}$$

In accordance with (17) the deflection $w(\xi)$ consists of waves travelling to the right and evanescent modes generated at $x = l_{i-1}$ and waves travelling to the left and evanescent modes generated at $x = l_i$. It is sufficient to consider just one term from (17) and to sum the result with respect to 'n' at the end of the calculations.

The index 'n' is omitted in this subsection. Hence we insert

$$w(\xi) = a_i e^{i\kappa_i(\xi - l_{i-1})} + b_i e^{i\kappa_i(\xi - l_i)}, \tag{21}$$

where the κ_i's have positive real values or positive imaginary parts. We insert this expression for $w(\xi)$ in (20) and carry out the integration with respect to ξ to obtain the following integrals in the complex k-plane,

$$I_i = \frac{K}{i}\{\mu_i - \mathcal{D}_i \kappa_i^4\} a_i \int_{\mathcal{L}'} \frac{\cosh kh}{k \sinh kh - K \cosh kh} \left\{ \frac{e^{ik(x-l_{i-1})} - e^{i\kappa_i(l_i - l_{i-1})} e^{ik(x-l_i)}}{\kappa_i - k} \right\} dk + \\ \frac{K}{i}\{\mu_i - \mathcal{D}_i \kappa_i^4\} b_i \int_{\mathcal{L}'} \frac{\cosh kh}{k \sinh kh - K \cosh kh} \left\{ \frac{e^{ik(x-l_i)} - e^{i\kappa_i(l_i - l_{i-1})} e^{ik(x-l_{i-1})}}{\kappa_i + k} \right\} dk. \tag{22}$$

One should realize that the values of κ_i still have to be chosen. We prefer to close the contours of integration in the complex plane and to apply the Cauchy residue theorem.

We first consider the cases $x < l_{i-1}$ and $x > l_i$. Here the Cauchy residue theorem can be applied directly. The Jordan lemma is valid for both integrals, either by closing the contour in the lower half-plane or the upper half-plane; see Figure 2. Because the expressions in { } in both integrals in (22) are not singular, it is clear that the only poles are the zeros of the dispersion relation for the water free surface, $k \tanh(kh) = K$. This dispersion relation has two real zeros $k = \pm k_0$ and infinitely many imaginary zeros $k = \pm k_j = \pm i\nu_j$. Depending on the position of x, we obtain different results for the integrals in (22).

– For $x < l_{i-1}$ both integrals are closed in the lower half-plane; see Figure 2. The contributions of the poles $-k_j$ for $j = 0, 1, \ldots, N-1$ are

$$2\pi K\{\mu_i - \mathcal{D}_i \kappa_{i,n}^4\} a_{i,n} \sum_{j=0}^{N-1} \frac{k_j}{k_j^2 h + K - K^2 h} \left(\frac{e^{-ik_j(x-l_{i-1})} - e^{i\kappa_{i,n}(l_i - l_{i-1})} e^{-ik_j(x-l_i)}}{\kappa_{i,n} - k_j} \right) + \\ 2\pi K\{\mu_i - \mathcal{D}_i \kappa_{i,n}^4\} b_{i,n} \sum_{j=0}^{N-1} \frac{k_j}{k_j^2 h + K - K^2 h} \left(\frac{e^{-ik_j(x-l_i)} - e^{i\kappa_{i,n}(l_i - l_{i-1})} e^{-ik_j(x-l_{i-1})}}{\kappa_{i,n} + k_j} \right). \tag{23}$$

They consist of one wave travelling to the left and $N-1$ evanescent modes generated at $x = l_{i-1}$.

– For $x > l_i$ both contours are closed in the upper half-plane, see Figure 2, so we get

$$2\pi K\{\mu_i - \mathcal{D}_i \kappa_i^4\} a_{i,n} \sum_{j=0}^{N-1} \frac{k_j}{k_j^2 h + K - K^2 h} \left(\frac{e^{ik_j(x-l_{i-1})} - e^{i\kappa_{i,n}(l_i - l_{i-1})} e^{ik_j(x-l_i)}}{\kappa_{i,n} - k_j} \right) + \\ 2\pi K\{\mu_i - \mathcal{D}_i \kappa_{i,n}^4\} b_{i,n} \sum_{j=0}^{N-1} \frac{k_j}{k_j^2 h + K - K^2 h} \left(\frac{e^{ik_j(x-l_{i-1})} - e^{i\kappa_{i,n}(l_i - l_{i-1})} e^{ik_j(x-l_{i-1})}}{\kappa_{i,n} + k_j} \right). \tag{24}$$

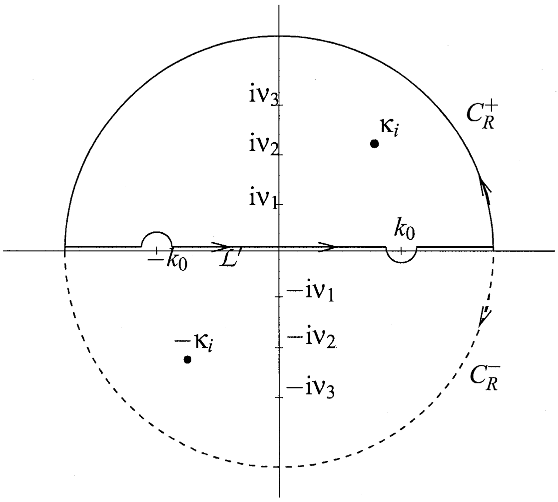


Figure 2. Closure of the contour of integration.

Here we have a wave travelling to the right and evanescent modes generated at $x = l_i$ Next we consider the case $l_{i-1} < x < l_i$. In this case the Cauchy residue theorem cannot be applied directly to the two integrals in (22) because the Jordan lemma is not valid for them. In order to apply the residue theorem, each integral in (22) is divided into two parts. In this way we arrive at four integrals, each of them having poles at the zeros of the water dispersion relation $k = \pm k_0$, $k = \pm \mathrm{i}\nu_j$.

Moreover, the first and second integrals have poles at $k = \kappa_i$ and the third and fourth integrals one at $k = \kappa_i$. The four new integrals have more poles than the two original integrals; however, the new integrals are suitable for their analytical evaluations by means of the residue theorem. The integrals are rather similar to each other, which is why the calculations of the first one are only discussed here. The value of the integral

$$I_{i1} = \int_{\mathcal{L}'} \frac{\cosh kh}{k \sinh kh - K \cosh kh} \frac{\mathrm{e}^{\mathrm{i}k(x-l_{i-1})}}{k_i - k} \mathrm{d}k \qquad (25)$$

depends on the sign of the imaginary part of κ_i and of the sign of $x - l_{i-1}$. We have chosen the imaginary part of κ_i to be positive, because of the choice of physical modes in (17). It is clear that for $x - l_{i-1} > 0$ this integral can be closed in the upper half-plane. We obtain contributions of the zeros of the water dispersion relation and of $k = \kappa_i$.

We first consider the contribution of the poles $k = \kappa_i$ to Equation (15). Comparing the coefficients of the exponential term $\mathrm{e}^{\mathrm{i}\kappa_i(x-l_{i-1})}$, we obtain the desired dispersion relation of the plate (18). The third integral also has to be closed in the upper half-plane. Here we only get contributions of the zeros of the water dispersion relation. The second integral can be closed in the lower half-plane. This also leads to contributions of the zeros of the water dispersion relation only, while the closure of the fourth integral in the lower half-plane leads to a contribution of $k = -\kappa_i$ and the ones of the water dispersion relation. In combination with (15) again the desired dispersion relation at the plate (18) is obtained.

From now on **the values of** κ_i **are known** (can be computed). The wetted-plate dispersion relation (18) for the *i*-th strip has two real zeros $\kappa_i = \pm\kappa_{i,0}$, four complex zeros $\kappa_i = \pm\kappa_{i,1} = \pm(\sigma_{i,1} + \mathrm{i}\rho_{i,1})$, $\kappa_i = \pm\kappa_{i,2} = \pm(-\sigma_{i,1} + \mathrm{i}\rho_{i,1})$ and infinitely many imaginary zeros $\kappa_i = \pm\kappa_{i,n} =$

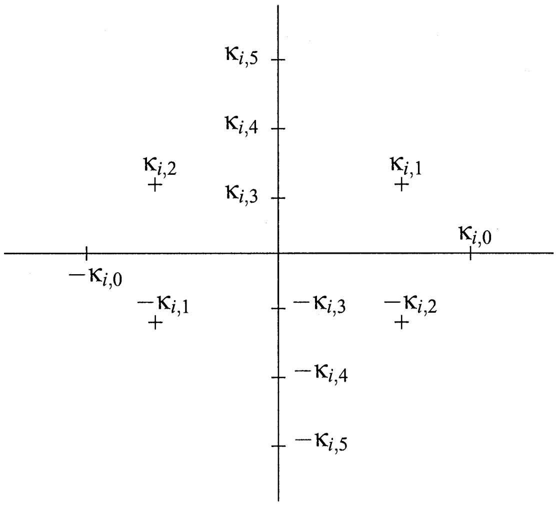


Figure 3. Zeros of the dispersion relation of the *i*-th wetted strip.

$\pm i\rho_{i,n}$ for $n = 3, 4, \ldots$; see Figure 3. As explained before, we choose the values along the positive real axis and in the upper half-plane. Hence we take $\kappa_i = \kappa_{i,n}$ for $n = 0, 1, \ldots, N+1$.

– Collecting the contributions of the residues of the zeros of the water dispersion relation, we obtain for $l_{i-1} < x < l_i$

$$2\pi K\{\mu_i - \mathcal{D}_i\kappa_{i,n}^4\}a_{i,n}\sum_{j=0}^{N-1}\frac{k_j}{k_j^2h + K - K^2h}\left(\frac{e^{ik_j(x-l_{i-1})}}{\kappa_{i,n} - k_j} - \frac{e^{i\kappa_{i,n}(l_i-l_{i-1})}e^{-ik_j(x-l_i)}}{\kappa_{i,n} + k_j}\right) +$$
$$2\pi K\{\mu_i - \mathcal{D}_i\kappa_{i,n}^4\}b_{i,n}\sum_{j=0}^{N-1}\frac{k_j}{k_j^2h + K - K^2h}\left(\frac{e^{ik_j(x-l_{i-1})}}{\kappa_{i,n} - k_j} - \frac{e^{i\kappa_{i,n}(l_i-l_{i-1})}e^{ik_j(x-l_{i-1})}}{\kappa_{i,n} + k_j}\right). \tag{26}$$

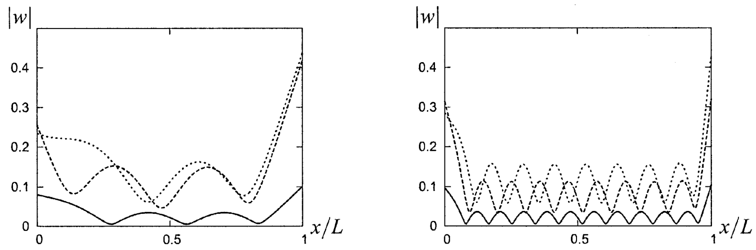
These contributions consist of waves travelling to the left and right, together with evanescent contributions generated at both sides of the interval.

We now consider the consequences of the contributions to the modes $e^{ik_j(x-l_{i-1})}$ and $e^{-ik_j(x-l_{i-1})}$ for $j = 0, 1, 2, \ldots$, for all the strips in the integral in Equation (15). The forcing at the right-hand side of (15) consists of one wave e^{ik_0x} travelling to the right. Hence, collecting all the contributions, we obtain an algebraic set of $I \times N$ equations for the modes travelling and evanescent to the right-hand side. For $j = 0, \ldots, N-1$ we have

$$\sum_{n=0}^{N+1}(\mu_1 - \mathcal{D}_1\kappa_{1,n}^4)\left[\frac{a_{1,n}}{\kappa_{1,n} - k_j} - \frac{b_{1,n}e^{i\kappa_{1,n}l_1}}{\kappa_{1,n} + k_j}\right] = \frac{k_0^2h + K - K^2h}{k_0K}\delta_j^0, \tag{27}$$

and for $m = 2, \ldots, I$ together with $j = 0, N-1$

$$\sum_{n=0}^{N+1}\sum_{i=1}^{m-1}(\mu_i - \mathcal{D}_i\kappa_{i,n}^4)\left\{a_{i,n}\left(\frac{e^{-ik_jl_{i-1}} - e^{i\kappa_{i,n}(l_i-l_{i-1})}e^{-ik_jl_i})}{\kappa_{i,n} - k_j}\right) + \right.$$
$$\left. b_{i,n}\left(\frac{e^{-ik_jl_i} - e^{i\kappa_{i,n}(l_i-l_{i-1})}e^{-ik_jl_{i-1}})}{\kappa_{i,n} + k_j}\right)\right\} + \tag{28}$$
$$\sum_{n=0}^{N+1}(\mu_m - \mathcal{D}_m\kappa_{m,n}^4)\left[\frac{a_{m,n}e^{-ik_jl_{m-1}}}{\kappa_{m,n} - k_j} - \frac{b_{m,n}e^{i\kappa_{m,n}(l_m-l_{m-1})}e^{-ik_jl_{m-1}}}{\kappa_{m,n} + k_j}\right] = \frac{k_0^2h + K - K^2h}{k_0K}\delta_j^0,$$



a. $L = 300$ m b. $L = 1000$ m

Figure 4. Deflection for $\Lambda = 150$, 90, 300 m. (top-down).

where δ_j^0 is the Kronecker delta function.

The modes travelling and evanescent to the left also give us $I \times N$ equations. Starting with the strip at the far end of the platform, we get for $j = 0, \dots, N-1$

$$\sum_{n=0}^{N+1} (\mu_I - \mathcal{D}_I \kappa_{I,n}^4) \left[\frac{-a_{I,n} e^{i\kappa_{I,n}(L - l_{I-1})}}{\kappa_{I,n} + k_j} + \frac{B_{I,n}}{\kappa_{I,n} - k_j} \right] = 0 \tag{29}$$

and for $m = 1, \dots, I-1$ together with $j = 0, \dots, N-1$

$$\begin{aligned} \sum_{n=0}^{N+1} \sum_{i=m+1}^{I} (\mu_i - \mathcal{D}_i \kappa_{i,n}^4) \Bigg\{ a_{i,n} \left(\frac{e^{ik_j l_{i-1}} - e^{i\kappa_{i,n}(l_i - l_{i-1})} e^{ik_j l_i}}{\kappa_{i,n} + k_j} \right) + \\ b_{i,n} \left(\frac{e^{ik_j l_i} - e^{i\kappa_{i,n}(l_i - l_{i-1})} e^{ik_j l_{i-1}}}{\kappa_{i,n} - k_j} \right) \Bigg\} + \\ \sum_{n=0}^{N+1} (\mu_m - \mathcal{D}_m \kappa_{m,n}^4) \left[\frac{-a_{m,n} e^{i\kappa_{m,n}(l_m - l_{m-1})} e^{ik_j l_m}}{\kappa_{m,n} + k_j} + \frac{b_{m,n} e^{ik_j l_m}}{\kappa_{m,n} - k_j} \right] = 0. \end{aligned} \tag{30}$$

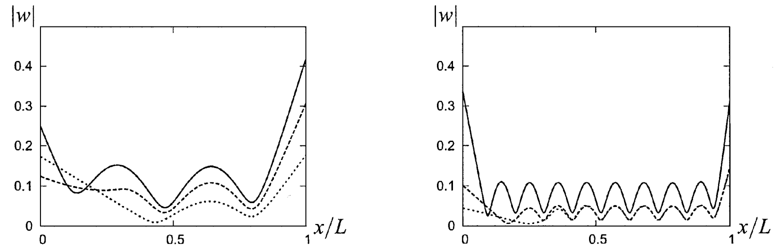
Combined with four boundary conditions at the outer edges and $4(I-1)$ matching conditions we have $2I(N+2)$ equations for the $2I(N+2)$ coefficients. The infinite system of algebraic equation is solved by truncation. The matrix is organised in such a way that the maximum elements are on its diagonal and that they can be made of the same order of magnitude. The amplitudes of the matrix elements decay exponentially with the distance from the main diagonal (in the case of finite water depth!). The system may be solved by an iteration method. However, the method developed is designed in such a way that the final system of equations for the coefficients $a_{i,n}$ and $b_{i,n}$ is 'very well organised' and its solution can be obtained accurately by most methods.

3.2. Reflection and transmission coefficients

The reflection and transmission coefficients follow from the expression of ϕ(x,0) as given in (12) and the definition of wave-height. In the far field the dimensionless free-surface elevation, with the notation $\zeta(x) = w(x)$, becomes

$$2\pi\zeta(x) = 2\pi e^{ik_0 x} + K \int_{\mathcal{P}} \mathcal{G}(x, 0; \xi, 0) \left\{ \frac{d^2}{d\xi^2} \mathcal{D}(\xi) \frac{d^2}{d\xi^2} - \mu(\xi) \right\} w(\xi) d\xi. \tag{31}$$

We insert the Green's function and notice that the poles $k = \pm k_0$ lead to the transmission and reflection coefficient, respectively. The evanescent modes are damped out in the far field.



a. $L = 300$ m and $L_1 = 30,\ 90,\ 150$ m b. $L = 1000$ m and $L_1 = 100,\ 200,\ 300$ m

Figure 5. Deflection for $\Lambda = 90$ m, $\mathcal{D}_1 L^4 = 10^{10}$ and $\mathcal{D}_2 L^4 = 10^7$.

The transmission coefficient becomes

$$T = 1 + \frac{k_0 K}{k_0^2 + K - K^2 h} \sum_{n=0}^{N+1} \sum_{i=0}^{I} (\mathcal{D}_i \kappa_{i,n}^4 - \mu_i) \left\{ a_{i,n} \left(\frac{e^{ik_0 l_{i-1}} - e^{i\kappa_{i,n}(l_i - l_{i-1})} e^{ik_0 l_i}}{\kappa_{i,n} - k_0} \right) + b_{i,n} \left(\frac{e^{ik_0 l_i} - e^{i\kappa_{i,n}(l_i - l_{i-1})} e^{ik_0 l_{i-1}}}{\kappa_{i,n} + k_0} \right) \right\}. \tag{32}$$

In a similar way we obtain the reflection coefficient

$$R = \frac{k_0 K}{k_0^2 + K - K^2 h} \sum_{n=0}^{N+1} \sum_{i=1}^{I} (\mathcal{D}_i \kappa_{i,n}^4 - \mu_i) \left\{ a_{i,n} \left(\frac{e^{ik_0 l_{i-1}} - e^{i\kappa_{i,n}(l_i - l_{i-1})} e^{ik_0 l_i}}{\kappa_{i,n} + k_0} \right) + b_{i,n} \left(\frac{e^{ik_0 l_i} - e^{i\kappa_{i,n}(l_i - l_{i-1})} e^{ik_0 l_{i-1}}}{\kappa_{i,n} - k_0} \right) \right\}. \tag{33}$$

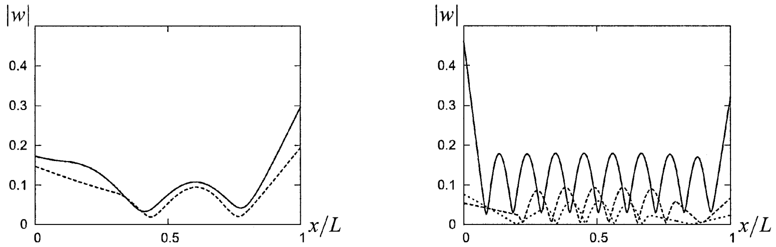
3.3. Numerical results

First we apply the new formulation to a single-strip problem, for which results were presented in [11]. The mass, per unit length, of the platform is kept constant in the next examples. In Figure 4 we show results for one strip with deep-water wavelength $\Lambda = 2\pi/K = 30$, 90, 150 m, $\mathcal{D}L^4 = 10^7$ and strip length $L = 300$, 1000 m, respectively. In all examples water depth $h = 10$ m. Comparison with [11] shows that the results are the same. The two actually solve the same problem.

We next consider the case that the platform consists of two connected strips. The total length is varied and the wavelength is fixed. We choose $L = 300$, 1000 m and $\Lambda = 90$ m. The lengths of the strips are varied. We consider $l_1/L = L_1 = 30$, 90, 300 m for the short one and $L_1 = 100$, 200, 300 m for the long one. The flexural rigidities $\mathcal{D}_1 L^4 = 10^{10}$ and $\mathcal{D}_2 L^4 = 10^7$ are chosen. The results are shown in Figure 5. If one compares Figure 4b and Figure 5b, one notices that for the case of a small value of L_1 a slight overall reduction of the deflection is obtained, except near $x = 0$ where the deflection increases. Recently this has also been observed by Eatock Taylor [14].

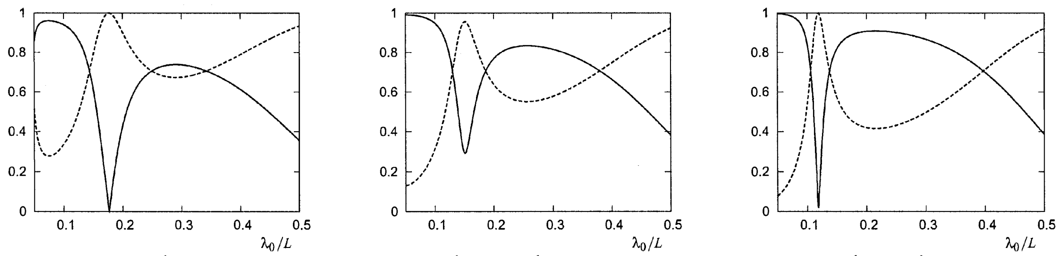
In Figure 6 results are shown for a platform consisting of three strips. We take at the two ends a strip with length $L_{1,3} = 100$, 200, 300 m and the same high value of the elastic rigidity.

The reflection and transmission coefficients are easily computed using (32) and (33). The Figures 7 show the reflection and transmission coefficients for platforms with total length $L = 300$ m. In Figure 7a the result is presented for the single strip with $\mathcal{D}L^4 = 10^7$, Figure 7b



a. $L = 300$ m and $L_{1,3} = 30,\ 90$ m   b. $L = 1000$ m and $L_{1,3} = 100,\ 200,\ 300$ m

Figure 6. Deflection for $\mathcal{D}_1 L^4 = 10^{10}$, $\mathcal{D}_2 L^4 = 10^7$ and $\mathcal{D}_3 L^4 = 10^{10}$.



a. one strip   b. two strips   c. three strips

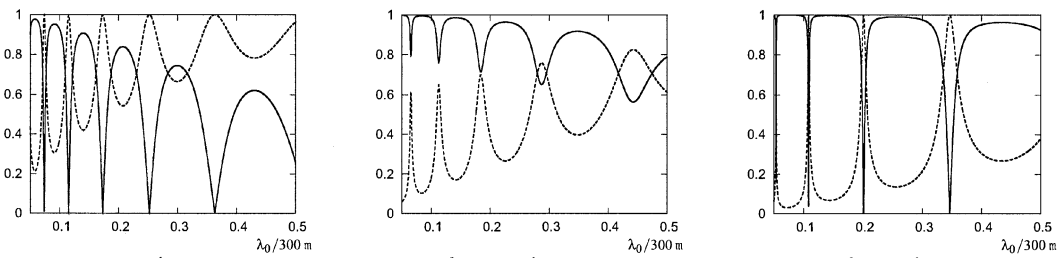
Figure 7. Reflection (——) and transmission (· · ·) coefficients for a platform of $L = 300$ m, $L_{1,3} = 90$ m).

shows the result for two strips with $L_1 = 90$ m, $\mathcal{D}_{1,2} L^4 = 10^{10,7}$ and in Figure 7c we have $L_{1,3} = 90$ m and $\mathcal{D}_{1,2,3} L^4 = 10^{10,7,10}$.

In Figure 8 we show results for three configurations with total length $L = 1000$ m and $L_{1,3} = 200$ m. The results indicate that the best results for the deflection are obtained in the situation of two strips.

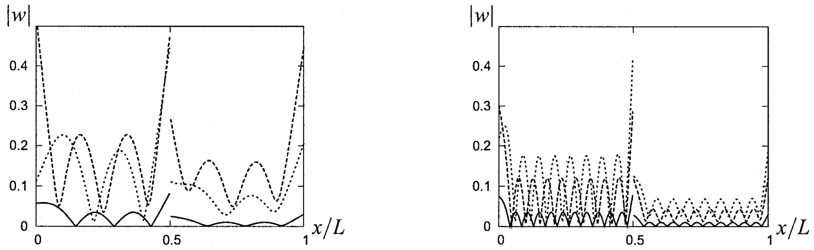
4. Disjoint strips

In this section we consider the case of two strips separated by a channel of water. For this situation the formulation does not differ much from the connected case, except that in the equation for the deflection (15) and the free-surface elevation outside the strips (31) the path of integration $\mathcal{P}$ consists of the two separated strips. The strips are force- and moment-free at the ends. Strip number one is extended, in dimensionless coordinates, from $x = 0$ to $x = l_1$



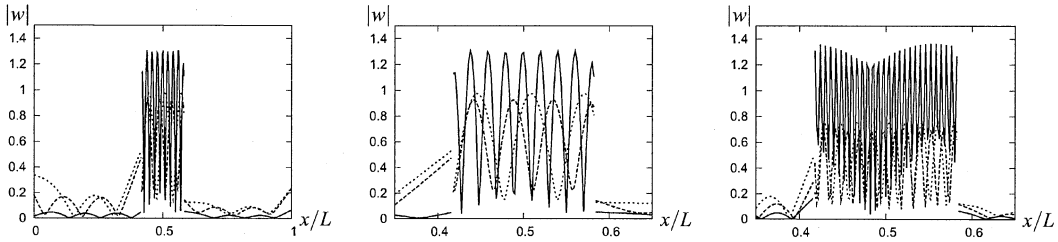
a. one strip   b. two strips   c. three strips

Figure 8. Reflection (——) and transmission (· · ·) coefficients for a platform of $L = 1000$ m, $L_{1,3} = 200$ m.



a. two strips with length 300 m, $L = 600$ m   b. two strips with length 1000 m, $L = 2000$ m

Figure 9. Deflection for disconnected strips for $\Lambda = 150$, 90, 30 m (top-down) and $\mathcal{D}L^4 = 10^7$.



a. two strips with length 300 m and gap 120 m   b. close-up   c. close-up of two strips with length 1000 m and gap 400 m

Figure 10. Amplitude of the platform deflection and of the waterelevation in the channel for $\Lambda = 150$, 90, 30 m and $\mathcal{D}L^4 = 10^7$.

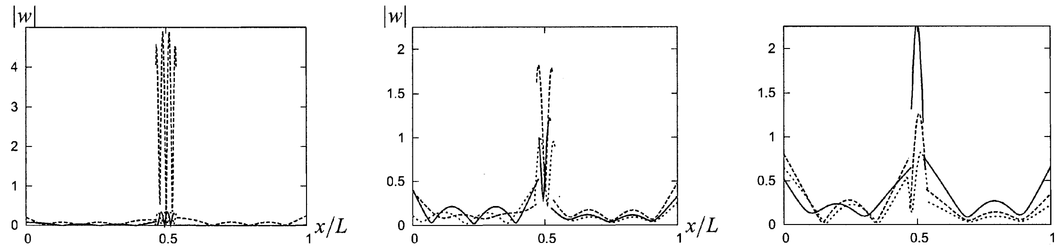
and the second $x = l_2$ to $x = 1$. The wave elevation in the channel now becomes

$$\begin{aligned}\zeta(x) = \mathrm{e}^{\mathrm{i}k_0 x} + \sum_{i=0}^{N-1} \frac{k_i K}{k_i^2 + K - K^2 h} \sum_{n=0}^{N+1} \\ (\mathcal{D}_1 \kappa_{1,n}^4 - \mu_1)\left\{a_{1,n}\left(\frac{\mathrm{e}^{\mathrm{i}k_i x} - \mathrm{e}^{\mathrm{i}\kappa_{1,n} l_1}\mathrm{e}^{\mathrm{i}k_i(x-l_1)}}{k_{1,n} - k_i}\right) + b_{1,n}\left(\frac{\mathrm{e}^{\mathrm{i}k_i(x-l_1)} - \mathrm{e}^{\mathrm{i}\kappa_{1,n} l_1}\mathrm{e}^{\mathrm{i}k_i,x}}{\kappa_{1,n} + k_i}\right)\right\} + \\ (\mathcal{D}_2 \kappa_{2,n}^4 - \mu_2)\left\{a_{2,n}\left(\frac{\mathrm{e}^{\mathrm{i}k_i(l_2-x)} - \mathrm{e}^{\mathrm{i}\kappa_{2,n}(1-l_2)}\mathrm{e}^{\mathrm{i}k_i(1-x)}}{k_{2,n} + k_i}\right) + b_{2,n}\left(\frac{\mathrm{e}^{\mathrm{i}k_i(1-x)} - \mathrm{e}^{\mathrm{i}\kappa_{2,n}(1-l_2)}\mathrm{e}^{\mathrm{i}k_i(l_2-x)}}{\kappa_{2,n} - k_i}\right)\right\}.\end{aligned} \tag{34}$$

First we show some results for the situation where two strips of equal length, are positioned such that the width of the water channel equals zero. In Figure 9 the results are given for $L = 600$, 2000 m and three values of the deep-water wavelength $\Lambda = 30$, 90, 150 m. Clearly the elevations at the free ends are discontinuous.

In Figure 10 the results for the amplitude of the deflection are shown for a channel between the plates. The width of the channel is 20% of L. In the gap the amplitude of wave-height is shown. The results converge very fast for increasing value of N.

We see that in some cases the amplitude of the wave elevation in the gap becomes large. This is due to resonance of the wave in the gap. This becomes more obvious if we vary the-size in the gap as can be seen in Figure 11 in the case of two strips with length 300 m, $\Lambda = 30$ m and three values of the width of the gap 40, 48 and 55 m and for $\Lambda = 90$ m and three values of the width of the gap 30, 40 and 50 m. It is clear that for certain frequencies resonance of the wave in the gap occurs. For $\Lambda = 30$ m we notice resonance among others for a spacing between the strips of about 48 m, while for $\Lambda = 90$ we have a peak at a spacing of about 40 m. In the case of two semi-infinite plates with a gap in between, Linton [3] also reported



a. gap —— 40, - - - 48, ... 55 m and $\Lambda = 30$ m
b. gap —— 30, - - - 40, ... 50 m and $\Lambda = 90$ m
c. same as b. with waterdepth $h = 100$ m instead of $h = 10$ m

Figure 11. Amplitude of the wave elevation in the gap between two strips of equal length 300 m with $\mathcal{D}L^4 = 10^7$.

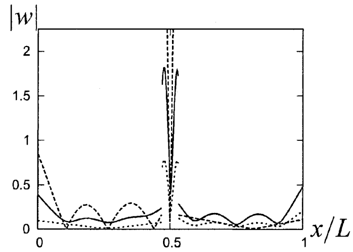


Figure 12. ___$\mathcal{D}_1 L^4 = \mathcal{D}_2 L^4 = 10^7$, - - - $\mathcal{D}_1 L^4 = 10^7$ and $\mathcal{D}_2 L^4 = 10^{10}$, ... $\mathcal{D}_1 L^4 = 10^{10}$ and $\mathcal{D}_2 L^4 = 10^7$.

resonance. Until now in all cases the value of the water depth was fixed at $h = 10$ m. In Figure 11c we show the effect of water depth in the same situation as in Figure 11a but with $h = 100$ m. One must realize that although the deep water wave length has been kept constant at 90 m, the wave length has been changed due to the change in depth. This can also be seen in the shift of the resonance peek. A further increase of the water depth does not change the picture visibly.

Next we choose one configuration and one wave-length to show (Figure 12) the infuence of different values of the rigidity parameter of the first strip or the second strip. We show results for the deep-water wave length $\Lambda = 90$ m, the length of the strips 300 m and the width of the gap 40 m. Compared with the case of equal strips the amplitude of the waves is reduced if the forward strip is more rigid than the second one and higher if they are reversed.

5. Conclusions and discussion

The computations shown in this paper indicate that the deflection of a flexible platform consisting of several strips with different constant physical coefficients can be described by means of the superposition of a finite number of travelling and evanescent modes. It turns out that for the water depth of 10 m and plate length of 300 m and more it is sufficient to take three modes at the plate into account. This means that in the water region all evanescent modes may be neglected. If the water depth becomes larger, truncation after more terms is necessary. Our experience is that in most cases ten terms are sufficient. Due to the structure of the coefficients of the matrix equation, one can take as many strips as one wants. In principe, the analytic approach of Linton [2] may be applied. We leave this as an exercise, because it will not improve the final result.

In all the computations for the reflection and transmission coefficients energy is conserved up to at least seven decimals. The only test of accuracy carried out is a convergence test. No comparison with results obtained independently in the past could be made. In the case of the single-strip case it was shown in [12] that the results presented by Takagi [13] are very close to each other. For the multiple-strip case no such comparison could be made because we are not aware of results by other authors that are suitable for comparison.

Results of the disjoint strips show that the level of the free surface between the strips may become very large. This resonant behaviour is also reported in [5]. In principle, other boundary conditions at the connection of the edges of the strips may be considered. We reported results for the tightly connected and free edges. Other situations only give rise to small changes in the matrix equations. Only the equations due to the boundary conditions have to be changed. Because all these changes do not alter the major part of the equations, the computer program is easily adapted.

There still remains the question about the physical relevance of the solutions obtained here. The incident wave hits a plate of zero thickness. The solution obtained here is the one that obeys the *Kantenbedingung*. Physically this means that the edges of the strips do not absorb or generate wave energy. Mathematically this means that only weak singularities are allowed. This is precisely the class of solutions that can be obtained by the expansions described here. Also the fact that the horizontal *wave-drift force* acting on the platform can be expressed directly in terms of the refuction coefficient (see [11]) gives some insight in the problem actually solved. A direct pressure integration along the platform gives a zero mean force in a horizontal direction. This seems to contradict the nonzero wave-drift force originating from the energy conservation used to derive the relation with the reflection coefficient. Taking into account the jump in surface elevation at the edge leads, as Guéret [9] has shown, to a good description of the wave-drift force. The physical interpretation of this result is that we have solved the limiting case (draft small) of the diffraction of waves by a platform with vertical extension above the free surface with no waves splashing over the deck and completely wetted bottom. The numerical study by Greco, Landrini and Faltinsen [15], where both slamming and green water are taken into account, indicates that in that situation the physical phenomenon is much more complicated than the one covered in this paper.

6. Acknowledgement

The author is grateful to Ken Takagi for his suggestion, made at the 18th lntenational Workshop on Water Waves and Floating Bodies in Le Croisic, France, to study the discontinuous-parameter case.

References

1. C.C. Mei and J.L. Black, Scattering of surface waves by rectangular obstacles in waters of finite depth. *J. Fluid Mech.* 38 (1969) 499–511.
2. C.M. Linton, The finite dock problem. *Zeitschr. angew. Math. Physik (Z4MP)* 52 (2001) 640–656.
3. C.M. Linton and H. Chung, Reflection and trarismission at the ocean/sea-ice boundary. *Wave Motion* 38 (2003) 43–52 (1968).
4. D.V. Evans and T.V. Davies, Wave-ice interaction. *Rep. 1313. Davidson Lab - Stevens lnstitute of Technology, New Jersey*

5. H. Chung and C.M. Linton, Interaction between water waves and elastic plates: Using the residue calculus technique. In: A.H. Clément and P. Ferrant (eds), *Proceedings of the 18th Int. Workshop on Water Waves and Floating Bodies*. Le Croisic, France: Ecole Centrale de Nantes (2003) pp. 36–40.
6. D.V. Evans and R. Porter, Wave scattering by narrow cracks in ice sheets floating on water of finite depth. *J. Fluid Mech.* 484 (2003) 143–165.
7. V.A. Squire, J.P. Dugan, P. Wadhams, P.J. Rottier and A.K. Liu, Of ocean waves and sea ice. *Annu. Rev. Fluid Mech.* 27 (1995) 115–168.
8. A.J. Hermans, A boundary element method for the interaction of free-surface waves with a very large floating flexible platform. *J. Fluids Struct.* 14 (2000) 943–956.
9. R. Guéret, *Interaction of Free Surface Wave with Elastic and Air-cushion Platforms*. Doctoral thesis, TUDelft, ISBN 90-407-2357-5 (2002) 119 pages.
10. A.J. Hermans, A geometrical-optics approach for the deflection of a floating flexible platform. *Appl. Ocean Res.* 23 (2002) 269–276.
11. A.J. Hermans, Interaction of free-surface waves with a floating dock. *J. Engng. Math.* 45 (2003) 39–53.
12. A.J. Hermans, The ray method for the deflection of a floating flexible platform in short waves. *J. Fluids Structs.* 17 (2003) 593–602.
13. Ken Takagi, Kiyoshi Shimada and Tetseru Ikebuchi, An anti-motion device for a very large floating structure. *Marine Structs.* (2000) 421–463.
14. R. Eatock Taylor, Wet or dry modes in linear hydroelasticity – why modes. In: R. Eatock Taylor (ed.) *Proceedings of the third International Conference on Hydroelasticity in Marine Technology.* Oxford, UK: The University of Oxford (2003) pp. 239–250.
15. M. Greco, M. Landrini and O.M. Faltinsen, Slamming and water shipping on a VLFS with shallow draft. In: A.H. Clément and P. Ferrant (eds), *Proceedings of the 18th Int. Workshop on Water Waves and Floating Bodies*. Le Croisic, France: Ecole Centrale de Nantes (2003) pp. 73–77.